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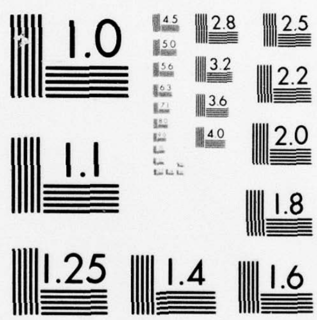

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20. Abstract The main algebraic and programming features of E.F.M. are illustrated by application to Laplace's equation for polygonal regions with elliptical cavities. The Basic Functions - Vertex, Edge and Mapped Polars - are obtained. Vertex equations, and Harmonic Fitting based on a discrete least squares criterion, are introduced.

Extension is made to a region sub-divided into a series of elements as in F.E. work. Major areas of F.E. work - Elastostatics, Thin Plates and Vibrations of Thin Elastic Shells - are sketched, and some basic problems for 3-D elasticity are solved.

The main features of E.F.M. are illustrated by numerical studies on the notch problem in Elastostatics and the vibration of a thin elastic shell with a square plan form.

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THE EDGE-FUNCTION METHOD (E.F.M.)

ABSTRACT OF INVITED LECTURE by P. M. QUINLAN to

The International Symposium on Innovative Numerical Analysis in Applied Engineering Science, Versailles, 23-27 May 1977.

(The Research was carried out in cooperation with Professor J. E. Fitzgerald, and Professor S. N. Atluri, both of Georgia Institute of Technology).

TEXT

SCOPE OF THE EDGE-FUNCTION METHOD

The Edge-Function Method as applied to two-dimensional problems is a procedure for obtaining exact solutions to acceptable Mathematical Models of practical boundary value problems in regular or irregular polygonal regions with, or without, cavities. The method was originally developed by Quinlan [1] for the torsion of prismatic bars of polygonal cross-section.

In the succeeding 12 years it has been successfully applied to problems in the bending of isotropic thin plates [2,3,4]; coupled linear systems in elastostatics [4,5,6,7] and in moderately thick plates and shallow shells [8]; cracks and stress concentrations in elastostatics [9]; vibrations of Thin Plates [10] and vibrations of Shallow Shells [11]. Selections from relevant abstracts are:

"A general computer method is presented of solving a very wide range of swept-back wing and skew bridge problems with any boundary conditions and loadings, with or without cut-outs and elastic column supports". [2]

"The Edge-Function Method may be described as a piecing together of "asymptotic" solutions to a set of linear p.d.e.'s for the several parts of a domain D to satisfy the boundary conditions in a discrete least squares sense. In the present paper, application is made to problems in elastostatics, where D may be multiply connected, bounded externally by an irregular polygon and with curved holes in the interior. The various "asymptotic" problems are solved for (1) the infinite sector, for Vertex Functions, (2) the semi-infinite region, leading to Edge-Functions, and (3) a curved hole in an infinite region, leading to Mapped Polar Biharmonics. [7]

"A new method for the numerical solution of Reissner-type plate and displacement-type shallow shell equations is developed. By deriving for an arbitrary edge of a polygonal plate or shallow spherical or rectangular cylindrical, hyperbolic paraboloidal, or elliptic paraboloidal shell a set of "edge-functions", it is possible to obtain accurate solutions to these structural components subjected to various support and load conditions". [8]

"The present investigation extends the method of Edge-Functions to the determination of natural frequencies and associated mode shapes of free vibration of thin elastic plates with a variety of boundary conditions. The Edge-Function technique essentially associates an independent coordinate system with every edge of the plate, and for every edge employs functions (satisfying the governing plate vibration equations) that rapidly decay with increasing distance from the plate boundary. Appropriate superposition of solutions stemming from all edges of the plate leads to an approximate representation of the time-dependent deflection surface". [10]

"It can confidently be concluded that the edge-function method, using Edge-Functions and the newly developed "Shell Polar" functions, provides a rapid means of determining accurate values for the frequencies of free vibration and mode shapes. It computes, as a routine, approximate root mean square boundary residuals for each boundary function, thus providing a practical measure for judging the acceptability of the solution proferred". [11]

FUNDAMENTALS OF E.F.M. VIA LAPLACE'S EQUATION

Section 2 of the paper aims to illustrate as simply as possible the main algebraic and programming features of E.F.M. by applying it to Laplace's equation, $\nabla^2 u = 0$, for polygonal regions with elliptical cavities.

On superposing I harmonic solutions, u_i, that represent the main characteristics of the problem, we obtain

$$u = \sum_{i=1}^{I} A_i u_i \; ; \quad \nabla^2 u_i = 0 \; , \tag{1}$$

where the I superposition constants A_i are evaluated numerically to satisfy, in an approximate way, the specified boundary conditions.

The method of harmonic fitting is developed, by expanding the boundary identity, say u=0, on a typical side j, $0<x'<a'$, in a Fourier sine series

$$0 \equiv u(x') = \sum_{N=1}^{\infty} C_N \sin nx' \; ; \quad C_N = \frac{2}{a'} \int_0^{a'} u(x') \sin nx'dx' \tag{2}$$

The series can, in practice, be truncated at N' terms giving the approximation

$$0 \equiv u(x') \approx \sum_{N=1}^{N'} C_N \sin nx' \; ; \quad 0 < x' < a' \; ; \quad n = N\pi/a' \tag{3}$$

provided that the series has convergence of order $1/N^3$ to ensure continuity of $u(x')$, which requires the two point, or vertex, equations

$$u(x') = 0 \; ; \quad x' = 0 \text{ and } x' = a' \tag{4}$$

Approximation (3) then leads to the harmonic equations

$$C_N = 0 \; ; \quad N = 1, \ldots . N' \tag{5}$$

On writing a pair of vertex equations and N' harmonic equations for each side of the polygon, a system of I linear equations results for the unknowns A_i, which equations may be written as

$$\tilde{K} \bar{A} = \bar{b} \; , \tag{6}$$

where $\tilde{K}$ is the coefficients matrix, and $\bar{b}$ the right hand column vector.

The integral for C_N in (2) may require numerical integration. A better approach is to interpret (3) as the fitting of a trigonometric expression to $u(x')$, and on using a discrete least squares criterion of fit, we obtain

$$C_N = \frac{2}{M'+1} \sum_{M'=1}^{M'} u(x'_M) \sin nx'_M \; ; \quad M' \geq N' \tag{7}$$

where x'_M are M' interior equidistant points on side x'. Note that the series for C_N is formally obtained by evaluating integral (3) numerically by the trapezoidal rule with M'+1 segments.

The basic problems - for the basic regions: infinite sector, semi-infinite strip and a hole in an infinite plane - encountered in putting together a suitable solution mix (1), are then solved. The resulting functions, called basic functions, are called the Vertex, Edge and Mapped Polar functions

respectively. Extension is made to mixed boundary conditions, and to polygons with elliptical indentations by the use of conformal mapping. A simple computer program is available to illustrate the basic features of E.F.M. for $\nabla^2 u = 0$ for a polygonal region, when u is specified on all boundary segments.

Fourier Cosine, and full Fourier series, are developed as alternatives to expansion (2), as these are required when dealing with Elastostatics and Orthotropic Thin Plates. Finally extension is made to a region divided into a series of elements, as in Finite Element work, and to cases where singularities occur either within or on the boundaries of the region.

A comprehensive computer program for Laplace/Poisson problems is available with illustrative examples, to act as a program prototype for the E.F.M. when applied to other areas - elastostatics, thin plates, vibrations etc.

Recent Research A largely self-contained paper [9] is due for publication. This sets the basis for dealing with coupled linear systems through the medium of elastostatics, and follows the general lines of section 1 to obtain the basic functions. Examples are given involving fracture and reentrant angles and a comprehensive program is available.

A companion work [14] on isotropic plates updates and extends considerably the earlier work [2], while [15] a Ph.D. thesis, to be represented in June, does a comprehensive job on aelotropic and orthotropic thin plates for polygonal regions with or without elliptic holes. This latter presents a general method for determining vertex functions directly from their eigenvalue determinant.

THREE-DIMENSIONAL ELASTICITY

Section 2 is a progress report on the development of E.F.M. for three-dimensional isotropic elasticity. The Papkovich-Neuber potentials [15] for the class of displacements

$$\bar{s} = \bar{B} + \nabla X \; ; \; \nabla^2 \bar{B} = 0, \tag{8}$$

are the key to obtaining (1) Plane Functions that decay exponentially inwards from each plane boundary area, and (2) Wedge Functions that model the singular behaviour between two intersecting plane faces - the respective analoges of the Edge and Vertex Functions in [9].

The boundary conditions on each plane face (x',y') lead, analogous to (2), to a plane boundary identity

$$\psi\,(x',y') = 0, \tag{9}$$

and a discrete least-squares surface fitting criterion gives a double series for harmonic fitting, analogous to (7). Thus the functions and methods for 2-D have their counterparts in 3-D and comparative results are expected before the Symposium for the problems solved by Deak [16] using Boundary Integral.

The coefficients matrix $\tilde{K}$ arising in (6) can be decomposed into

$$\tilde{K}_1 A = b - \tilde{K}_2 \bar{A}, \tag{10}$$

where, since the terms $\tilde{K}_2$ are of minor significance, their effects can be obtained by iteration on the banded matrix for kth-iteration:

$$\tilde{K}_1 A = \bar{b}^{(k)} \tag{11}$$

Usually three iterations provide an acceptable solution, the time spent on the iterations being only a few percent of that required to solve the banded matrix K_1. Thus E.F.M. is now firmly within the banded matrix field, and is capable of dealing with any number of connected elements as in Finite Element work.

MAIN ADVANTAGES

The effective algorithmic method that has been developed for controlling the algebra [9], allied to a systems approach adopted to programming, facilitates the extension of E.F.M. into new areas.

Solutions for up to four different sizes of matrix are computed as a routine. Each solution vector provides an exact solution to the specified

problem but with slightly different boundary values, which are computed as a routine, the differences being termed the boundary residuals. If the residuals are within the limits within which an engineer can specify the boundary values, then E.F.M. offers as "exact" a solution as can be obtained to the problem.

Derived quantities e.g. moments, shears, are obtained as accurately as the primary quantity, and are quickly computed for several truncation levels.

Perhaps the greatest advantage of E.F.M. is that no pre-computer processing of the problem is required. Only the geometrical, load and material data are required and an E.F.M. program can be operated as a "black box" without the user having any knowledge of E.F.M.

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